

Infinite Circuits in Descriptive Set Theory

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Montréal DDC Seminar, 2026

Separating low levels of the Borel hierarchy:

Introduction

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$\Sigma_1^0 \neq \Pi_1^0$: Basic topology

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Using universal sets and diagonalization:

Theorem (Borel Hierarchy)

We have $\Sigma_\alpha^0 \neq \Pi_\alpha^0$ for all $1 \leq \alpha < \omega_1$.

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- 1 Infinite circuits
- 2 A warm-up proof
- 3 Borel hierarchy theorem via circuits
- 4 Conclusion

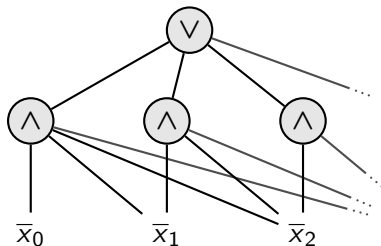
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What is an infinite circuit?

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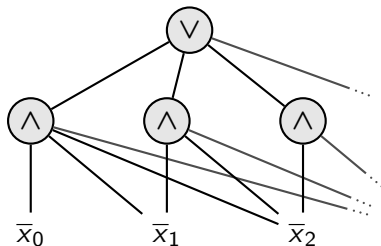
An example of an infinite circuit.

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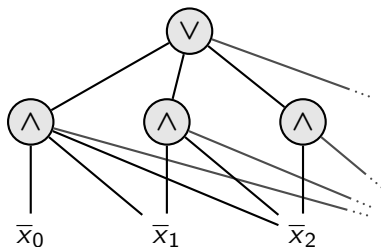
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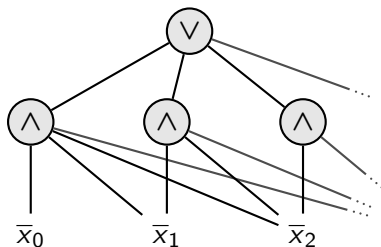
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One of the vertices is designated as the *output node*.

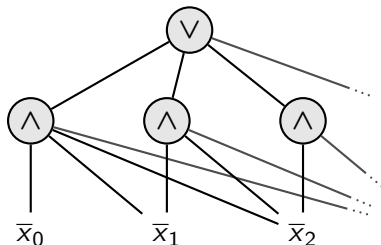


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Evaluating circuits

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Given some $x \in 2^{\mathbb{N}}$ and circuit C , we define the *evaluation* of each node in C inductively:



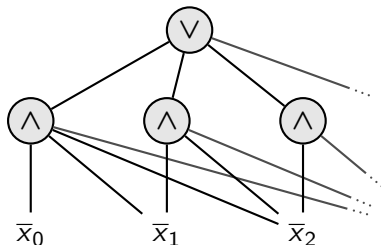
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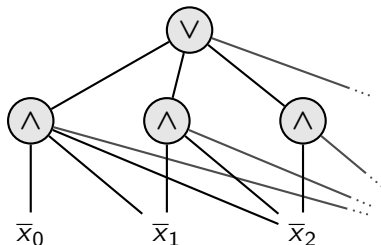
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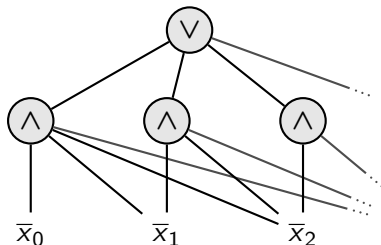
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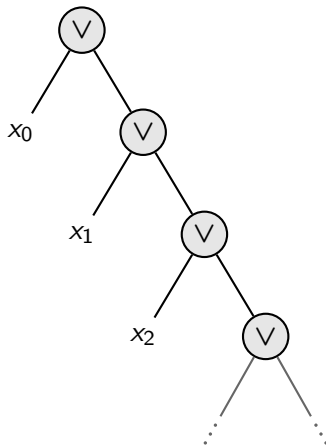
We define $C(x)$ to be the evaluation of the output node of C . A circuit C therefore *computes* a function $C: 2^{\mathbb{N}} \rightarrow \{0, 1\}$.



An example of an infinite circuit.

Another example

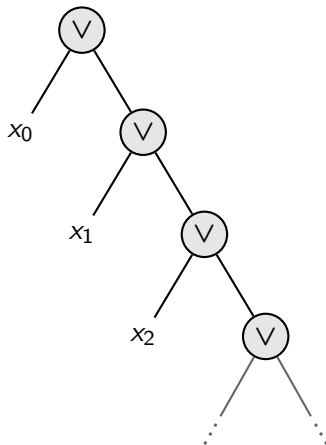
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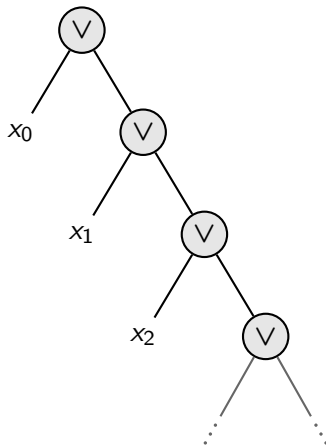
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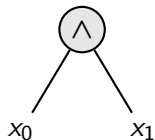


Not well-founded!

Circuit rank

Definition

A circuit is Σ_0 or Π_0 if it is finite.



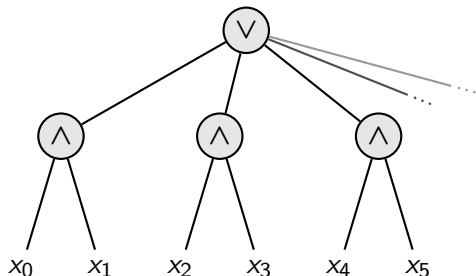
An example of a Σ_0/Π_0 -circuit

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A Σ_α -circuit is a countable disjunction of circuits C_n , where each C_n is a Π_{β_n} -circuit for some $\beta_n < \alpha$.



An example of a Σ_1 -circuit

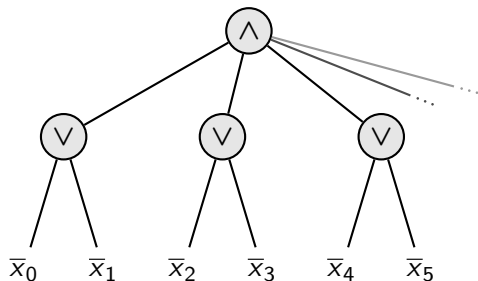
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Every circuit has rank α for some $\alpha < \omega_1$.

Lemma

A set $S \subseteq 2^{\mathbb{N}}$ is clopen if and only if $\mathbb{1}_S$ is computed by a finite circuit.

Circuits and Borel sets

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Corollary

A set $S \subseteq 2^{\mathbb{N}}$ is Borel if and only if $\mathbb{1}_S$ is circuit-computable.

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Analytic sets

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A set $A \subseteq 2^{\mathbb{N}}$ is *analytic* if there exists a Borel set $B \subseteq 2^{\mathbb{N}} \times 2^{\mathbb{N}}$ such that

$$x \in A \iff \exists y \in 2^{\mathbb{N}} \text{ such that } (x, y) \in B.$$

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Theorem

A set $A \subseteq 2^{\mathbb{N}}$ is analytic if and only if there exists a G_{δ} (i.e. Π_2^0) set $G \subseteq 2^{\mathbb{N}} \times 2^{\mathbb{N}}$ such that

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Projections of Borel sets $\iff$ Projections of G_δ sets!

Reframing via circuits

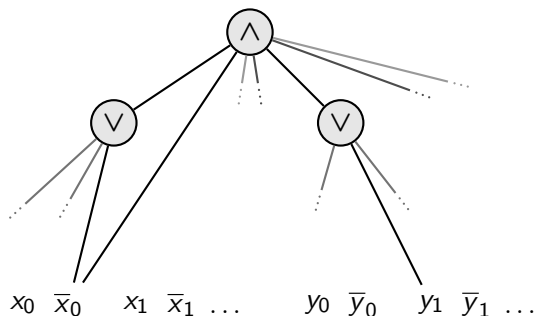
Proof. If A is analytic, then there exists a circuit C such that

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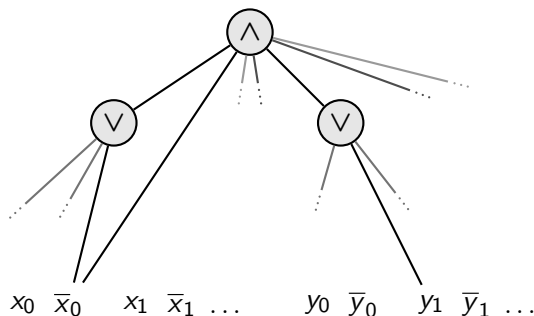
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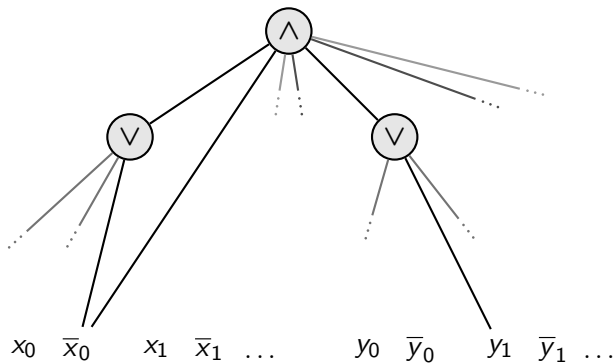
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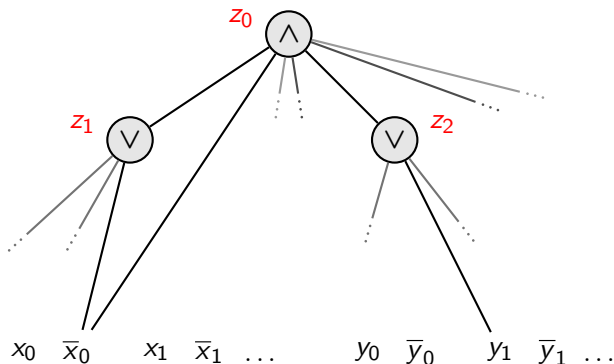
We call this a *nondeterministic circuit*.

Shifting work to the prover



Problem: The depth of C may be greater than 2.

Shifting work to the prover



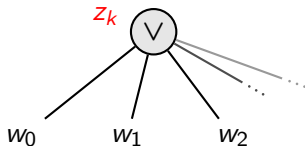
Problem: The depth of C may be greater than 2.

Idea: Make the prover evaluate C for us!

Checking validity

Suppose z_k represents an $\vee$ -gate.

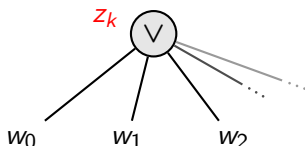
Each w_j is in $\{x_i, \bar{x}_i, y_i, \bar{y}_i, z_i, \bar{z}_i\}$
for some $i \in \mathbb{N}$.



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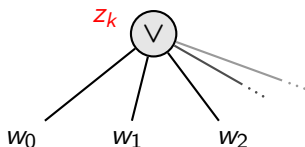
Then z_k is valid if and only if

$$z_k \implies \bigvee_{j \in \mathbb{N}} w_j \quad \text{and} \quad \bar{z}_k \implies \bigwedge_{j \in \mathbb{N}} \bar{w}_j.$$

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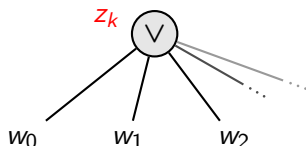
Equivalently:

$$\bar{z}_k \vee \left(\bigvee_{j \in \mathbb{N}} w_j \right) \quad \text{and} \quad \bigwedge_{j \in \mathbb{N}} (z_k \vee \bar{w}_j).$$

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Equivalently:

$$\underbrace{\bar{z}_k \vee \left(\bigvee_{j \in \mathbb{N}} w_j \right)}_{\Sigma_1\text{-circuit!}} \quad \text{and} \quad \bigwedge_{j \in \mathbb{N}} \underbrace{(z_k \vee \bar{w}_j)}_{\Sigma_1\text{-circuit!}}.$$

Putting everything together

Take the conjunction of all clauses from the preceding slide for each $k \in \mathbb{N}$, along with the clause z_0 . Let C' denote the resulting Π_2 -circuit.

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which means that

$$\begin{aligned} x \in A &\iff \exists y \in 2^{\mathbb{N}} \text{ such that } C(x, y) = 1 \\ &\iff \exists y, z \in 2^{\mathbb{N}} \text{ such that } C'(x, y, z) = 1. \end{aligned}$$

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So A is the projection of a G_δ set!



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Reframing via circuits

We will begin with the finite-rank version:

Theorem

We have for all $n \in \mathbb{N}$ that $\Sigma_n^0 \subsetneq \Sigma_{n+1}^0$.

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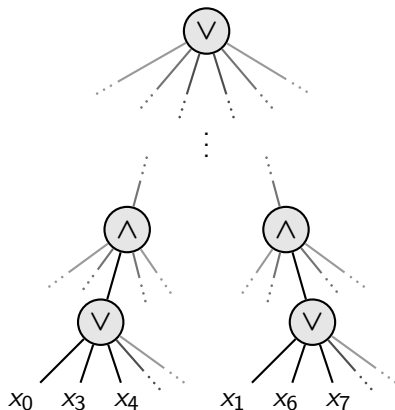
In the language of circuits:

Theorem (Sipser, 1983)

For any $n \in \mathbb{N}$, there exists a function $f: 2^{\mathbb{N}} \rightarrow \{0, 1\}$ which is computed by a Σ_{n+1} -circuit but not by a Σ_n -circuit.

Setup of the proof

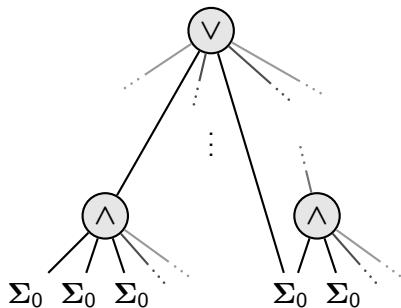
Proof.



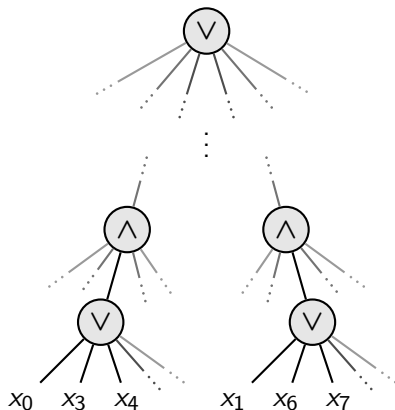
A full alternating tree T_{n+1}

Setup of the proof

Proof.



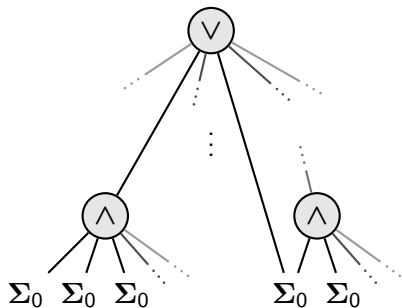
An arbitrary Σ_n -circuit C



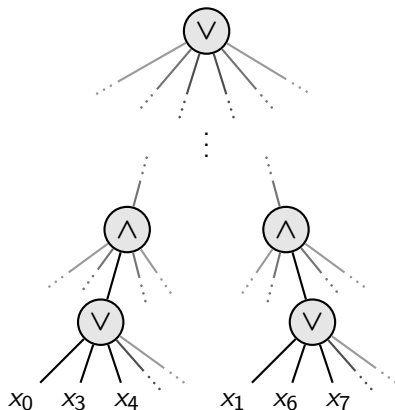
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Proof.



An arbitrary Σ_n -circuit C



A full alternating tree T_{n+1}

Goal: Show that C and T_{n+1} compute different functions.

Restrictions

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A *restriction* is a partial setting of the input variables $x_0, x_1, x_2, \dots$ to 0 or 1.

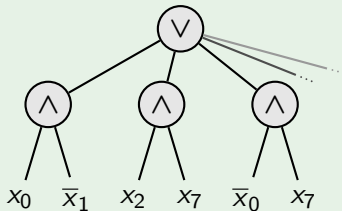
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Example

Every non-constant Σ_1 -circuit can be forced to 1 by assigning finitely many variables.



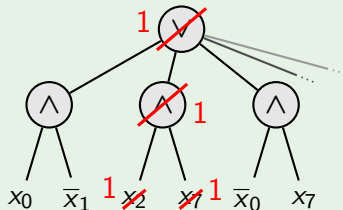
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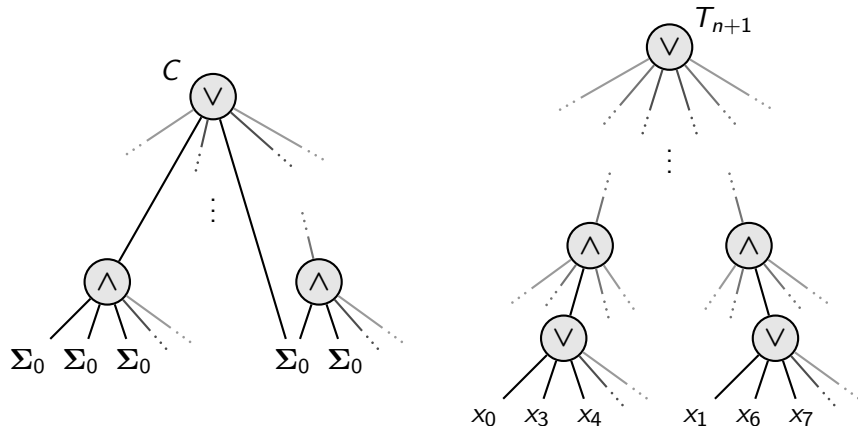
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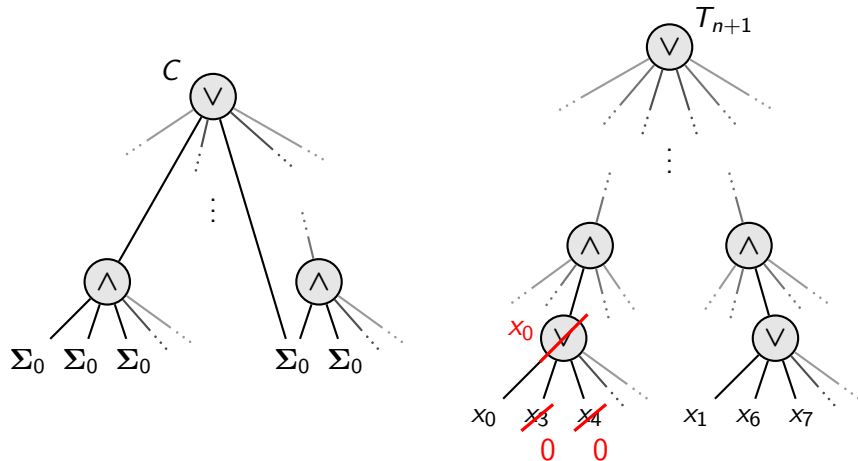


Constructing a restriction



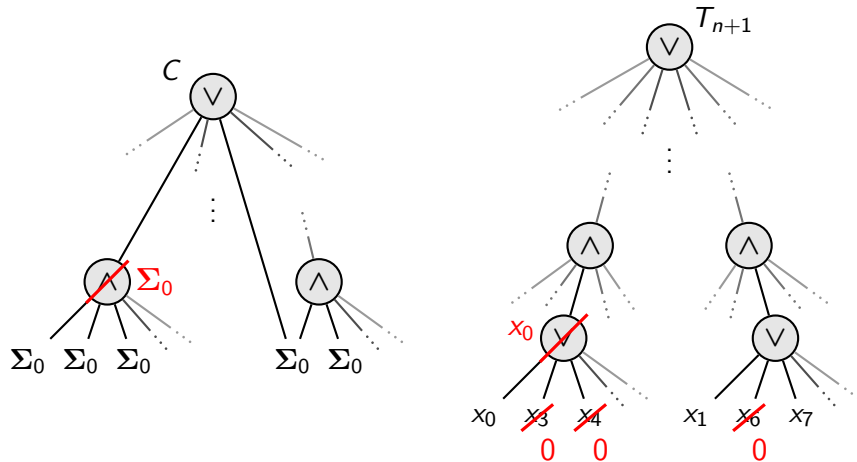
Goal: Find a restriction ρ such that C^ρ is a Σ_{n-1} circuit while T_{n+1}^ρ computes a full alternating tree of rank n .

Constructing a restriction



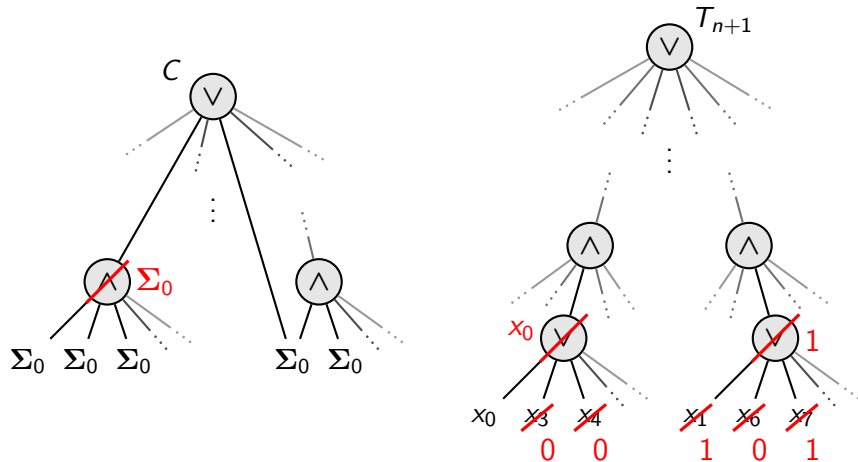
Step 1: Simplify a single Σ_1 -circuit in T_{n+1} . Mark one variable (x_0 in this case) as *preserved*.

Constructing a restriction



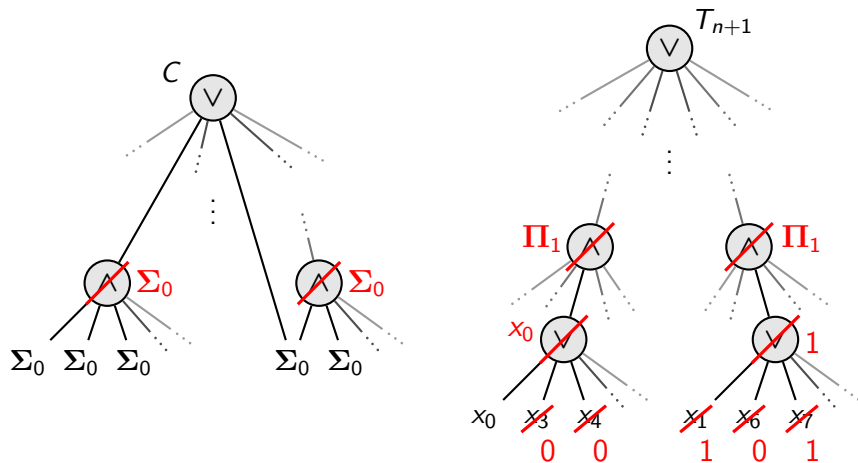
Step 2: Simplify a single Π_1 -circuit in C (setting finitely many variables to 0 or 1).

Constructing a restriction



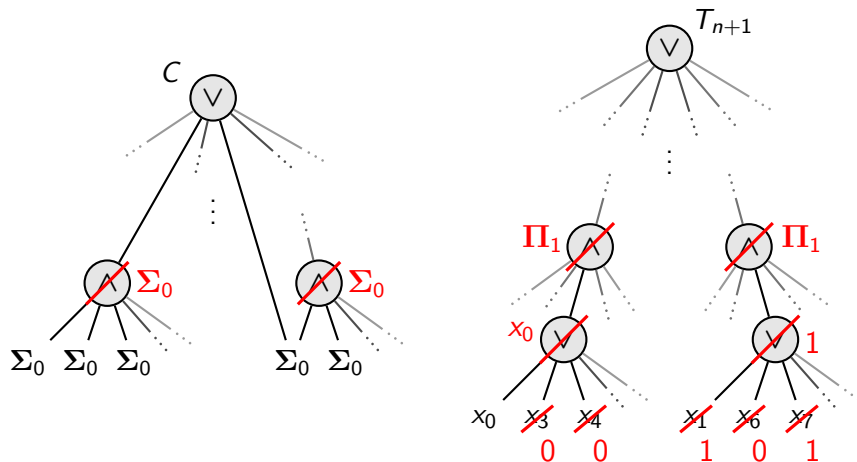
Step 3: Clean up by *suppressing* finitely many Σ_1 -circuits in T_{n+1} (which we can now ignore).

Constructing a restriction



Step 4: Repeat, making sure every Π_2 -circuit in T_{n+1} gets infinitely many preserved variables.

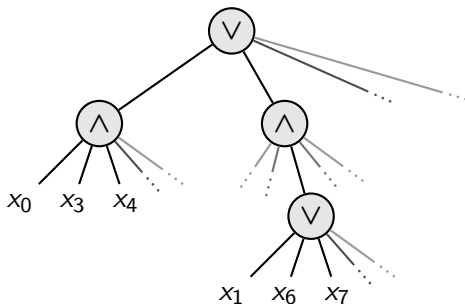
Constructing a restriction



Conclusion: C^ρ is a Σ_{n-1} -circuit, while T_{n+1}^ρ is a full alternating tree of rank n . □

Extending to infinite rank

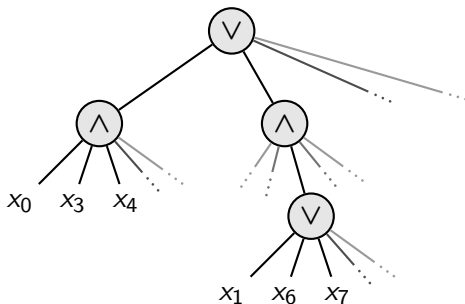
We can construct full alternating trees with rank α for any $\alpha < \omega_1$.



A full alternating tree of rank ω

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A full alternating tree of rank ω

Can we show that no circuit with rank $\beta < \alpha$ computes the same function as T_α ?

Challenges with infinite rank

Some problems:

Replacing rank-1 subcircuits with finite circuits need not reduce the complexity!

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We also need a new construction at limit stages.

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Other results

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Conjecture

$P \neq NP$?

Thank You!

Questions?